# MATH 4001 – Real Analysis III

YEAR 2016-2017

## Instructor's Information

**Instructor:** Paul Skoufranis
**E-mail:** pskoufra at yorku.ca
**Office:** Ross Building, South 625
**Offices Hours:** Tuesdays from 10:00AM to 12:00PM, and by appointment

## Administrative Information

**Course Prerequisites:** SC/MATH 3210, or both SC/MATH 3001 and SC/MATH 2310
**Course Webpage:** http://pskoufra.info.yorku.ca/teaching/y2016-math4001/
**Lectures:** Tuesdays and Thursdays from 4:30PM to 6:00PM in North Ross 501
**Textbook:** *Real Analysis,* Cambridge University Press, N.L. Carothers (recommended)
**Test Dates:** Test One → Tuesday October 25, 2016, in class
Test Two → Tuesday January 10, 2017, in class
Test Three → Tuesday February 28, 2017, in class
**Final Examination Dates:** TBA. The final examination will be comprehensive and will be scheduled by the registrar during the April exam period.

## Course Description and Objectives

In previous calculus courses, students have been introduced to many interesting concepts, such as convergence sequences, infinite sums, continuous functions, derivatives, and Riemann integrals. These concepts, although incredible useful with applications throughout mathematics, have some shortcomings. For example, several natural functions are not Riemann integrable, and the notions of convergence, summability, and continuity have only been defined on the real numbers. This course will focus on extending these concepts to a more general context thereby permitting a wider notion of real analysis.

Roughly this course will be divided into two parts. In the first part, we will analyze which mathematical structures have notions of convergence sequences and continuous functions. For a sequence to converge, it must get 'close' to its limit and, for the real numbers, we defined 'close' by using a notion of distance. Consequently, we will define a metric space to be a set together with a distance function. This will permit a comprehensive study of convergent sequences and continuous functions. The notions of a complete space and a compact space, which are vital in the study of analysis, will also be discussed. The abstract theorems we will obtain have applications in applied mathematics, such as differential equation theory. Finally, we will be able to determine when certain continuous functions on the real numbers uniformly approximate other continuous functions.

In the second part of the course, we will improve the notion of the integral. The Lebesgue integral will remove most of the shortcomings of the Riemann integral and many important integration theorems will be discussed. Combined with the first part of the course, this will permit a study of Fourier analysis on a closed interval. One of the biggest pitfalls in analysis was the assumption that the Fourier series of a function converges uniformly to the function. Thus we will spend significant time studying convergence of Fourier series. Time permitting, we will also discuss other notions related to the Lebesgue integral, such as derivatives.

## Course Schedule

The following is a rough outline of material that will be covered in the lectures of this course:

1. Basic Set Theory
2. Cardinality
    a. Equivalence Relations and Partial Orders
    b. Cantor-Schröder-Bernstein Theorem
    c. Zorn's Lemma
3. Metric Spaces
    a. Normed Linear Spaces
    b. Topology
    c. Continuity
4. Completeness
    a. Banach Spaces
    b. Cantor's Theorem
5. Banach Space Theorems
    a. The Baire Category Theorem
    b. The Open Mapping Theorem
    c. The Closed Graph Theorem
    d. The Principle of Uniform Boundedness
    e. The Banach Contractive Mapping Theorem
6. Hilbert Spaces
7. Compact Metric Spaces
    a. The Heine-Borel Theorem
    b. The Finite Intersection Property
    c. The Borel-Lebesgue Theorem
    d. Finite Dimensional Normed Linear Spaces
    e. Arzela-Ascoli Theorem
8. Approximating Continuous Functions
    a. The Weierstrass Approximation Theorem
    b. The Stone-Weierstrass Theorem
9. Measure Spaces
    a. The Lebesgue Measure
10. The Lebesgue Integral
    a. The Monotone Convergence Theorem
    b. Fatou's Lemma
    c. The Dominated Convergence Theorem
11. Fourier Series
    a. The Riemann-Lebesgue Lemma
12. Convergence of Fourier Series
13. Derivatives of Measure
    a. The Riemann-Stieltjes Integral
    b. Absolute Continuity
    c. Bounded Variation
    d. The Lebesgue Differentiation Theorem

## Marking Scheme

A student's final grade in the course will be computed as follows:

25% Homework Assignments + 45% Tests (15% Each) + 30% Final Examination

There will be approximately 10 homework assignments during the course. The lowest assignment score will be dropped and the remaining will be weighted equally.

## Homework Assignments

The purpose of the homework in this course is to aid students in the comprehension of the material presented in lecture each week and to expand students' knowledge beyond what can be covered in lectures. Thus the instructor will endeavour to provide students with a sufficient amount of time after the material is presented in lecture for completion of the homework.

Homework will be posted on the course webpage and students will have approximately two weeks to complete assignments. Homework will be due in class on the due date and late homework will not be accepted, as solutions will be posted promptly. Students are expected to clearly indicate their names and student ID number on their homework.

Students are welcome to collaborate with each other on the homework. However, each student must write his or her solutions separately in their own words (no copying!).

## Regrading

A student that believes there has been an error in the grading of their work should bring it to the attention of the instructor within two weeks from the time at which the work was returned to the class. Objections that arise after this two-week period will not be considered.

## Make-up Policy

If you know you will miss a test/assignment for a valid excuse (e.g. religious holiday, university sanctioned event, etc.), please contact the course instructor at least a week prior to the absence so alternate accommodations can be made. If you missed a test/assignment due to a valid medical emergency, please contact the course instructor directly. Late assignments will be accepted and make-up tests will be arranged only if accompanied by a note from a medical professional (http://mech.lassonde.yorku.ca/wp-content/uploads/2015/10/attend_physician_statement.pdf).

If you missed a final exam due to a valid medical emergency, please follow the instructions for Deferred Exam Procedures (http://www.registrar.yorku.ca/exams/deferred/). Download the forms for Deferred Standing and the Attending Physician's Statement, and submit the completed forms to the undergraduate office no later than 5 business days from the date of the exam. Once the forms have been approved, students will be emailed the decision regarding the deferred status. Students should also notify their instructor that they did not write the exam and explain why.

## Academic Integrity

York students are required to maintain the highest standards of academic honesty and they are subject to the Senate Policy on Academic Honesty (http://secretariat-policies.info.yorku.ca/policies/academic-honesty-senate-policy-on/). The policy affirms the responsibility of faculty members to foster acceptable standards of academic conduct and of the student to abide by such standards. Students are expected to review the materials on the Academic Integrity website (http://www.yorku.ca/academicintegrity).

## Learning Disability Services

York University has policies in place to ensure that all students have an equal opportunity to attain their educational goals. Accommodations related to diagnosed learning disabilities may be made through Learning Disability Services. If you would like confidential support or academic accommodations, please visit http://lds.info.yorku.ca.

## Accessibility for Persons with Disabilities

The York University Accessibility Hub (http://accessibilityhub.info.yorku.ca/) is your online stop for accessibility on campus. The Accessibility Hub provides tools, assistance and resources.