Comprehensive Exam Syllabus: Measure Theory

Graduate Program in Mathematics and Statistics, York University

FALL 2017

Topics

The Measure Theory Comprehensive Examination during the Fall 2017 semester may cover the following topics:

1. Measures

   Algebras, σ-algebras, Borel sets, measures, σ-finite measures, outer measures, Catathéodory's method, Catathéodory-Hahn Extension Theorem, Lebesgue-Steiltjes measures, regularity of measures, metric outer measures(*), Hausdorff measures and dimension(*)
2. Measurable Functions

   Measurable functions, approximation by simple functions, Egoroff's Theorem, Lusin's Theorem
3. Integration over Measure Spaces

   Integral of complex measurable functions, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem, L_p-Spaces, Holder's Inequality
4. Differentiation and Integration

   Vitali Covering Lemma, Lebesgue Differentiation Theorem, functions of bounded variation, absolutely continuous functions, Cantor's ternary function, The Fundamental Theorems of Calculus
5. Signed Measures

   Signed measures, absolutely continuous measures, mutually singular measures, Hahn Decomposition Theorem, Jordan Decomposition Theorem, Radon-Nikodym Theorem, Lebesgue Decomposition Theorem
6. Product Measures and Fubini's Theorem
7. Riesz Representation Theorems (*)

Topics marked with a (*) will only be included if sufficient time in MATH 6280 is found to cover these topics.

Textbook

The reference textbook for this examination is *Real Analysis* by Royden and Fitzpatrick (4th edition). Students attempting this comprehensive examination should be familiar with the topics listed above as presented in Chapters 1-7, 17-20. Alternatively, students can view the course notes and assignments for MATH 6280 to be found at http://pskoufra.info.yorku.ca/teaching/f2017-math6280/.

Grading

The three-hour comprehensive examination will consist of 8 questions each worth 10 points. A student is required to obtain a score of 48 points or greater (i.e. 60%) to pass the comprehensive examination. For students enrolled in MATH 6280, the final examination component of their grade will be computed as a score out of 60.