

MATH 6280 – Measure Theory

FALL 2019

Instructor's Information

Instructor: Paul Skoufranis

E-mail: pskoufra at yorku.ca

Office: Ross Building, South 625

Offices Hours: Tuesdays 8:45AM-9:45AM and 3:30PM-4:30PM
Thursdays 2:30PM-3:30PM
By appointment

Administrative Information

Course Prerequisites: Undergraduate Real Analysis.

Course Webpage: <http://pskoufra.info.yorku.ca/teaching/f2019-math-6280/>

Lectures: Tuesdays and Thursdays from 1:00PM to 2:30PM in VH 2009.

Textbook: *Real Analysis*, by Royden and Fitzpatrick, 4th edition (optional).

Midterm Examination Date: Thursday October 31st, 2019.

Final Examination Date: TBA. The final examination will be comprehensive and will be during the December exam period. The final examination will be a closed book examination.

Comprehensive Examination

An extended version of the final examination of MATH 6280 will serve as a comprehensive examination for the PhD program in the Department of Mathematics and Statistics. A detailed syllabus of the material and format of the comprehensive examination can be found on the course webpage. Students who are not enrolled in MATH 6280 yet wish to take the comprehensive examination should contact the instructor if they require more detail pertaining to the comprehensive examination.

Course Description and Objectives

A measure on a collection of subsets (or events) of a set is a function with certain desirable properties that describes the size (or probability) of each element of the collection. Given a measure it is possible to define an integral of specific functions against the measure thereby generalizing the Riemann integral. Both measures and this integration theory are essential concepts in probability theory and functional analysis.

In this course, we will begin by definition the notion of a measure and developing methods for constructing measures. Subsequently, we will study specific functions, known as measurable functions, and prove results such as Lusin's Theorem and Egorff's Theorem. Measurable functions will permit an integration theory and the study of many additional topics, such as Fatou's Lemma, the Dominated Convergence Theorem, Fubini's Theorem, Hahn and Jordan decompositions, the Lebesgue Differentiation Theorem, the Fundamental Theorems of Calculus, the Radon-Nikodym Theorem, the Lebesgue Decomposition Theorem, and L_p -spaces.

Course Schedule

The following is a rough outline of material that will be covered in the lectures of this course:

1. Measures
 - a. The Caratheodory Method
 - b. Extending Measures
 - c. Lebesgue-Stieljes Measures
 - d. Metric Outer Measures (*)
 - e. Hausdorff Measures (*)
2. Measurable Functions
 - a. Lusin's Theorem
 - b. Egoroff's Theorem
3. Integration over Measure Spaces
 - a. Integral of Complex Measurable Functions
 - b. Monotone Convergence Theorem
 - c. Fatou's Lemma
 - d. Dominated Convergence Theorem
 - e. Lp-Spaces
4. Differentiation and Integration
 - a. Vitali Covering Lemma
 - b. Lebesgue Differentiation Theorem
 - c. Functions of Bounded Variation
 - d. Absolutely Continuous Functions
 - e. The Fundamental Theorems of Calculus
5. Signed Measures
 - a. Hahn Decomposition Theorem
 - b. Jordan Decomposition Theorem
 - c. Radon-Nikodym Theorem
 - d. Lebesgue Decomposition Theorem
6. Product Measures and Fubini's Theorem
7. Riesz Representation Theorems (*)

(*) indicates time permitting.

Marking Scheme

A student's final grade in the course will be computed as follows:

$$30\% \text{ Homework Assignments} + 25\% \text{ Midterm Examination} + 45\% \text{ Final Examination}$$

There will be approximately 6 homework assignments during the course due approximately every two weeks.

Homework Assignments

The purpose of the homework in this course is to aid students in the comprehension of the material presented in lecture each week and to expand students' knowledge beyond what can be covered in lectures. Thus the instructor will endeavour to provide students with a sufficient amount of time after the material is presented in lecture for completion of the homework.

Homework will be posted on the course webpage and students will have approximately two weeks to complete assignments. Homework will be due in class on the due date and late homework will not be accepted as solutions will be posted promptly. Students are expected to clearly indicate their names and student ID number on their homework.

Students are welcome to collaborate with each other on the homework. However, each student must write his or her solutions separately in their own words (no copying!).

Regrading

A student that believes there has been an error in the grading of their work should bring it to the attention of the instructor within two weeks from the time at which the work was returned to the class. Objections that arise after this two-week period will not be considered.

Missed Assessment Policy

If you have serious issues pertaining to the examination dates (e.g. religious holiday), please contact the professor immediately. There will be no make-up assessments for assignments in this course, nor will there be any extensions. If you know you will miss an examination for a valid excuse (e.g. religious holiday, university sanctioned event, etc.), please contact the course instructor at least a week prior to the absence so alternate accommodations can be made. If you missed an examination due to a valid medical emergency, please contact the course instructor directly and immediately. Make-up examinations will be arranged only if accompanied by a note from a medical professional (http://mech.lassonde.yorku.ca/wp-content/uploads/2015/10/attend_physician_statement.pdf).

If you missed the final exam due to a valid medical emergency, please follow the instructions for Deferred Exam Procedures (<http://myacademicrecord.students.yorku.ca/deferred-standing>). Download the forms for Deferred Standing and the Attending Physician's Statement, and submit the completed forms to the undergraduate office no later than 5 business days from the date of the exam. Once the forms have been approved, students will be emailed the decision regarding the deferred status. Students should also notify their instructor that they did not write the exam and explain why.

Academic Integrity

York students are required to maintain the highest standards of academic honesty and they are subject to the Senate Policy on Academic Honesty (<http://secretariat-policies.info.yorku.ca/policies/academic-honesty-senate-policy-on/>). The policy affirms the responsibility of faculty members to foster acceptable standards of academic conduct and of the student to abide by such standards. Students are expected to review the materials on the Academic Integrity website (<https://spark.library.yorku.ca/academic-integrity-what-is-academic-integrity/>).

Numerous students in Faculty of Science courses have been charged with academic misconduct when materials they uploaded to third party repository sites (e.g. Course Hero, One Class, etc.) were taken and used by unknown students in later offerings of the course. The Faculty's Committee on Examinations and Academic Standards (CEAS) found in these cases that the burden of proof in a charge of aiding and abetting had been met, since the uploading students had been found in all cases to be wilfully blind to the reasonable likelihood of supporting plagiarism in this manner. Accordingly, to avoid this risk, students are urged not to upload their work to these sites. Whenever a student submits work obtained through

Course Hero or One Class, the submitting student will be charged with plagiarism and the uploading student will be charged with aiding and abetting.

Note also that exams, tests, and other assignments are the copyrighted works of the professor assigning them, whether copyright is overtly claimed or not (i.e. whether the © is used or not). Scanning these documents constitutes copying, which is a breach of Canadian copyright law, and the breach is aggravated when scans are shared or uploaded to third party repository sites.

Student Accessibility Services

York University has policies in place to ensure that all students have an equal opportunity to attain their educational goals. Accommodations related to diagnosed learning disabilities may be made through Student Accessibility Services. If you would like confidential support or academic accommodations, please visit <https://accessibility.students.yorku.ca/>.

Mental Health

York University aims to promote a healthy, inclusive, and supportive environment that fosters mental health and well being. For more information or if you are struggling with mental health, please see <http://mhw.info.yorku.ca/>.