

# MATH 6280 – Measure Theory

FALL 2018

## Instructor's Information

**Instructor:** Paul Skoufranis

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**Office:** Ross Building, South 625

**Offices Hours:** Tuesdays 9:30AM-10:30AM and 4:00PM-5:00PM, Thursdays 9:30AM-10:30AM, and by appointment

**Math Lab Hours:** Tuesdays and Thursdays 10:30AM-11:30AM

## Administrative Information

**Course Prerequisites:** Undergraduate Real Analysis.

**Course Webpage:** <http://pskoufra.info.yorku.ca/teaching/f2018-math6280/>

**Lectures:** Tuesdays and Thursdays from 1:00PM to 2:30PM in CB (Chemistry) 122.

**Textbook:** *Real Analysis*, by Royden and Fitzpatrick, 4<sup>th</sup> edition (optional).

**Midterm Examination Date:** In class on Tuesday October 30th, 2018.

**Final Examination Date:** TBA. The final examination will be comprehensive and will be scheduled during the December exam period in consultation with students. The final examination will be a closed book examination.

## Comprehensive Examination

An extended version of the final examination of MATH 6280 will serve as a comprehensive examination for the PhD program in the Department of Mathematics and Statistics. More information pertaining to the comprehensive examination will be distributed to the students closer to the examination date. Students who are not enrolled in MATH 6280 yet wish to take the comprehensive examination should contact the instructor regarding the time, place, and material for the examination.

## Course Description and Objectives

A measure on a collection of subsets (or events) of a set is a function with certain desirable properties that describes the size (or probability) of each element of the collection. Given a measure it is possible to define an integral of specific functions against the measure thereby generalizing the Riemann integral. Both measures and this integration theory are essential concepts in probability theory and functional analysis.

In this course, we will begin by defining the notion of a measure and developing methods for constructing measures. Subsequently, we will study specific functions, known as measurable functions, and prove results such as Lusin's Theorem and Egorff's Theorem. Measurable functions will permit an integration theory and the study of many additional topics, such as Fatou's Lemma, the Dominated Convergence Theorem, Fubini's Theorem, Hahn and Jordan decompositions, the Lebesgue Differentiation Theorem, the Fundamental Theorems of Calculus, the Radon-Nikodym Theorem, the Lebesgue Decomposition Theorem, and  $L_p$ -spaces.

## **Course Schedule**

The following is a rough outline of material that will be covered in the lectures of this course:

1. Measures
  - a. The Caratheodory Method
  - b. Extending Measures
  - c. Lebesgue-Stieljes Measures
  - d. Metric Outer Measures (\*)
  - e. Hausdorff Measures (\*)
2. Measurable Functions
  - a. Lusin's Theorem
  - b. Egoroff's Theorem
3. Integration over Measure Spaces
  - a. Integral of Complex Measurable Functions
  - b. Monotone Convergence Theorem
  - c. Fatou's Lemma
  - d. Dominated Convergence Theorem
  - e.  $L_p$ -Spaces
4. Differentiation and Integration
  - a. Vitali Covering Lemma
  - b. Lebesgue Differentiation Theorem
  - c. Functions of Bounded Variation
  - d. Absolutely Continuous Functions
  - e. The Fundamental Theorems of Calculus
5. Signed Measures
  - a. Hahn Decomposition Theorem
  - b. Jordan Decomposition Theorem
  - c. Radon-Nikodym Theorem
  - d. Lebesgue Decomposition Theorem
6. Product Measures and Fubini's Theorem
7. Riesz Representation Theorems (\*)

(\*) indicates time permitting.

## **Marking Scheme**

A student's final grade in the course will be computed as follows:

$$30\% \text{ Homework Assignments} + 25\% \text{ Midterm Examination} + 45\% \text{ Final Examination}$$

There will be approximately 6 homework assignments during the course due approximately every two weeks.

## **Homework Assignments**

The purpose of the homework in this course is to aid students in the comprehension of the material presented in lecture each week and to expand students' knowledge beyond what can be covered in lectures. Thus the instructor will endeavour to provide students with a sufficient amount of time after the material is presented in lecture for completion of the homework.

Homework will be posted on the course webpage and students will have approximately two weeks to complete assignments. Homework will be due in class on the due date and late homework will not be accepted, as solutions will be posted promptly. Students are expected to clearly indicate their names and student ID number on their homework.

Students are welcome to collaborate with each other on the homework. However, each student must write his or her solutions separately in their own words (no copying!).

## **Regrading**

A student that believes there has been an error in the grading of their work should bring it to the attention of the instructor within two weeks from the time at which the work was returned to the class. Objections that arise after this two-week period will not be considered.

## **Make-up Policy**

If you know you will miss a test/assignment for a valid excuse (e.g. religious holiday, university sanctioned event, etc.), please contact the course instructor at least a week prior to the absence so alternate accommodations can be made. If you missed a test/assignment due to a valid medical emergency, please contact the course instructor directly. Late assignments will be accepted and make-up tests will be arranged only if accompanied by a note from a medical professional ([http://mech.lassonde.yorku.ca/wp-content/uploads/2015/10/attend\\_physician\\_statement.pdf](http://mech.lassonde.yorku.ca/wp-content/uploads/2015/10/attend_physician_statement.pdf)).

If you missed a final exam due to a valid medical emergency, please follow the instructions for Deferred Exam Procedures (<http://myacademicrecord.students.yorku.ca/deferred-standing>). Download the forms for Deferred Standing and the Attending Physician's Statement, and submit the completed forms to the undergraduate office no later than 5 business days from the date of the exam. Once the forms have been approved, students will be emailed the decision regarding the deferred status. Students should also notify their instructor that they did not write the exam and explain why.

If you require a religious accommodation for the final examination, please follow the procedures at <https://w2prod.sis.yorku.ca/Apps/WebObjects/cdm.woa/wa/regobs>.

## **Academic Integrity**

York students are required to maintain the highest standards of academic honesty and they are subject to the Senate Policy on Academic Honesty (<http://secretariat-policies.info.yorku.ca/policies/academic-honesty-senate-policy-on/>). The policy affirms the responsibility of faculty members to foster acceptable standards of academic conduct and of the student to abide by such standards. Students are expected to review the materials on the Academic Integrity website (<https://spark.library.yorku.ca/academic-integrity-what-is-academic-integrity/>).

## **Learning Disability Services**

York University has policies in place to ensure that all students have an equal opportunity to attain their educational goals. Accommodations related to diagnosed learning disabilities may be made through Learning Disability Services. If you would like confidential support or academic accommodations, please visit <http://lds.info.yorku.ca>.

## **Accessibility for Persons with Disabilities**

The York University Accessibility Hub (<http://accessibilityhub.info.yorku.ca/>) is your online stop for accessibility on campus. The Accessibility Hub provides tools, assistance and resources.

## **Mental Health**

York University aims to promote a healthy, inclusive, and supportive environment that fosters mental health and well-being. For more information or if you are struggling with mental health, please see <http://mhw.info.yorku.ca/>.

## **Intellectual Property**

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