YorkU Putnam Training **Week 2: Pigenholes, Invariants, and Extremes**

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Important Concepts

1. **Pigenhole Principle**
   - The Pigenhole principle states that if m and n are natural numbers with $m > n$, then if m objects (e.g. pigeons) are placed into n containers (e.g. pigeon holes), then at least one container must contain more than one item.
   - Useful if you want to show that at least two numbers are equal when you have a list of m numbers from $\{1, \ldots, n\}$ with $m > n$.
   - Useful if you want to show that at least two objects have the same property out of a list of properties.
   - Useful in the following context: Given natural numbers m and n with $m > n$ and integers $\{a_1, \ldots, a_m\}$, there exists $i < j$ and k such that $a_j = a_i + kn$.
2. **Invariants**
   - The idea is to pick an expression that is invariant (i.e. does not change) as something is done. This means that after a process is completed, the value expression can be determined.
   - Some common invariants are: odd or even, the value of k such that a number is of the form $mn + k$ for some previously fixed n.
3. **Extremes**
   - Pick an extreme and see what happens.
   - Usually best coupled with trying to obtain a contradiction: if this extreme case happens then we get a contradiction.
   - The extreme is often a best or worse case, or a maximal or minimal value.