

EXAMPLE PROBLEMS

Question 1. Show that some pair of any 5 points in the unit square will be at most $\frac{\sqrt{2}}{2}$ units apart.

Question 2. Show that at any party (with at least two people in attendance) there are two people who have the same number of friends at the party (assume that all friendships are mutual).

Question 3. Find the minimum number of breaks required to break an $m \times n$ bar of chocolate into 1×1 squares.

Question 4. In a ping pong tournament, each player plays every other player exactly once. Show that there is some player in the unfortunate position that every other player either beat him or beat someone who did.

PREVIOUS RELATED PUTNAM PROBLEMS

Question 1. (AB1) Given a positive integer n , what is the largest k such that the numbers $1, 2, \dots, n$ can be put into k boxes so that the sum of the numbers in each box is the same? [When $n = 8$, the example $\{1, 2, 3, 6\}, \{4, 8\}, \{5, 7\}$ shows that the largest k is *at least* 3.]

Question 2. (AB1) Let d_1, d_2, \dots, d_{12} be real numbers in the open interval $(1, 12)$. Show that there exists distinct indices i, j, k such that d_i, d_j , and d_k are the side lengths of an acute triangle.

Question 3. (AB1) Recall that a regular icosahedron is a convex polyhedron having 12 vertices and 20 faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is written a nonnegative integer such that the sum of all 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

Question 4. (AB1) Let \mathcal{P} denote the set of vectors defined by $\mathcal{P} = \{(a, b) \mid 0 \leq a \leq 2, 0 \leq b \leq 100, \text{ and } a, b \in \mathbb{Z}\}$. Find all $\vec{v} \in \mathcal{P}$ such that the set $\mathcal{P} \setminus \{\vec{v}\}$ can be partitioned into two sets of equal size and equal sum.

Question 5. (AB1) Let $S = \{1, 2, \dots, 9\}$. For a partition $\alpha = \{A_1, \dots, A_\ell\}$ of S (i.e. A_1, \dots, A_ℓ are disjoint subsets of S with union S) and an element $x \in S$, let $N(\alpha, x)$ be the number of elements in the set A_i which contains x . Show that for any two partitions α and β of S there exists two distinct elements $x, y \in S$ such that $N(\alpha, x) = N(\alpha, y)$ and $N(\beta, x) = N(\beta, y)$.

Question 6. (AB1) Let $n \in \mathbb{N}$ and for each $i \in \{1, \dots, n\}$ and $j \in \{1, 2, 3\}$ let $a_{i,j} \in \mathbb{Z}$. Suppose for each $i \in \{1, \dots, n\}$ that at least one of $a_{i,1}, a_{i,2}, a_{i,3}$ is odd. Show there exists some $x, y, z \in \mathbb{Z}$ such that for at least $\frac{4n}{7}$ values of i , the integer $a_{i,1}x + a_{i,2}y + a_{i,3}z$ is odd.

Question 7. (Modified AB3) There are 2021 boxes labelled $B_1, B_2, \dots, B_{2021}$, and $2021n$ balls have been distributed among them, for some positive integer n . You may redistribute the balls by a sequence of moves, each of which consists of choosing an i and moving *exactly* i balls from box B_i into any one other box. For which values of n is it possible to reach the distribution with exactly n balls in each box, regardless of the initial distribution of balls?